SS2 THIRD TERM MATHEMATICS LESSON NOTE

WEEK 1

TOPIC: CHORD PROPERTY

CONTENT:

- **Lines and regions of a circle.**
- **Circle theorems including:**
- **Angles subtended by chords in circle;**
- **Angles subtended by chords at the centre;**
- **Perpendicular bisectors of chords;**
- **Angles in alternate segments.**
- **Cyclic quadrilaterals**

ANGLES SUBTENDED BY CHORDS IN CIRCLE

The word chord is a straight line joining any two points such as A and B on the circumference of a circle. The chord divides the circle into two parts called the segments (minor and major)

Major Arc

major segment

chord

minor segment

Minor Arc

The larger part of the circle is called the major segment while the smaller part --- the minor segment. Each of these parts is called the alternate segment of the other.

Note: A major segment has a major arc while a minor segment a minor arc.

A circle is the set of all points at a constant distance from a fixed point in a plane. The fixed point is the centre of the circle, the distance from the fixed point (is constant), is called the radius.

It will be noted that it is the chord that subtends (project out) angles viz:

Q

P R

A B

From the diagram, P,Q and R are points on the circumference of a circle. are angles subtended at the circumference by the chord AB or by the minor arc AB. are all angles in the same major segment APQRB.

Similarly, from the diagram below

A B

X Y

.A are angles subtended by the chord AB or by the major arc AB in the minor segment AXYB or the alternate segment.

ANGLES SUBTENDED BY CHORDS AT THE CENTRE

Examples:

Theorem*: A straight line drawn from the centre of the circle to the middle point of a chord which is not a diameter, is at right angle*

O

A D B

Given: A chord AB of a circle with centre O, is the mid-point of AB such that AD = DB

To prove:

Construction: join OA and OB

Proof: (radii of the circle)

(Given)

is common

Hence

But

⇒

THEOREM: Equal chords of a circle are equidistant from the centre of the circle.

A D

M N

B C

Given: chord AB = chord DC

To prove:

Construction: join

Proof: In

OA = OD (radii)

Converse: chords that have the same distance (i.e equidistant) from the centre of the circle are of the same length. If , then

Examples:

A chord of length 24cm is 13cm from the centre of the circle. Calculate the radius of the circle

Solution:

P Q

From the diagram,

In

.

= 169 + 144

= 313

, r = = 17.69cm

Class Activity:

A chord is 5cm from the centre of a circle of diameter 26cm.Find the length of the chord. (WAEC)

Calculate the length of a chord which is 6cm from the centre of the circle of radius 10cm

PERPENDICULAR BISECTORS OF CHORDS

This talks of line(s) that divides another line into two equal parts.

THEOREM: A straight line drawn from the centre of a circle perpendicular to a chord bisects the chord.

A D B

Given: A chord AB of a circle with centre O and

To prove:

Construction: join OA and OB

Proof: In

(given)

OD is common

Examples;

1. **XYZ is an isosceles triangle inscribed in a circle centre O. XY = XZ = 20cm and YZ = 18cm. calculate to 3s.f**

The altitude of XYZ

The diameter of the circle

Solution: **X** **X**

A **B**

Y **Z** **Y** **Z**

In

.

(XQ) =

= 17.9cm

(b.) is the diameter of the circle , radii =

In ,

In

But diameter,

2.The diagram below shows two parallel chords AB and CD that lie on opposite sides of the centre O of the circle. AB = 40cm, CD = 30cm and the radius of the circle is 25cm. Calculate the distance h between the two chords

A E B

H O

C F D

Solution:

Similarly,

In by Pythagoras' theorem,

.

.

In

But,

Class Activity

A chord 26cm long is 10cm away from the centre of a circle. Find the radius of the circle.

The diameter of a circle is 12cm if a chord is 4cmfrom the centre, calculate the length of the chord.

ANGLES IN ALTERNATE SEGMENTS

Recall: The chord that passes through the centre of the circle is called diameter and is the largest chord in a circle.

A segment is a region bounded by a chord and an arc lying between the chord's end point.

The chord that is not a diameter divides the circle into two segments -- a major and a minor segment.

But, a tangent to a circle is a straight line that touches the circle at a point.

Thus;

Theorem*: An angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment*

D

E

C

B

P **A** **Q**

Given: A circle with tangent PAQ at A and chord AC dividing the circle into two segments AEC and ABC. Segments AEC is alternate to

To prove: <QAC = <AEC and <PAC

= <ABC

Construction: Draw the diameter AD. Join CD

Proof: From the lettering in the above,

Also, <ACD = 90 (angle in a semi-circle)

In

<ACD = 180 (sum of angles in a)

Subtracting from equations (i) and (ii)

Also, B is a point in the minor segment.

< PAC + < CAQ = 180 (angles on a straight line)

< PAC + = 180

< PAC = 180 –

= 180 (proved)

< PAC = < ABC (opposite angles of a cyclic quadrilateral)

Example:

is a tangent to circle QPS. Calculate < SQX

X

S Q

P Z

Solution:

In < SPQ = 180 – (55 + 48)

= 180 – 103

= 77

.< SQX = 77 (angles in alternate segment)

Example: N

Y

Z

L X M

From the above, are tangents to the circle with centre O. Find X

Solution:

<XYZ = 2 (angles in alternate segment)

<ZYN + 100 = 180 (angles on a straight line)

<ZXY = 80 (angles in alternate segment)

<XZY = 100 – 2x (angles in alternate segment)

<MXY = <XZY = 100 – 2x (angles in alternate segment)

.<MXY = <MYX =100 – 2x

. is an isosceles triangle

.35 + 2(100 –2x) = 180 (sum of angles in a)

2(100 –2x) = 180 – 35

200 – 4x = 145

4x = 55

.

Class Activity:

1. **PQ and PT are tangents to a circle with centre O. Find the unknown angles giving reasons.**

2. **PQ and PT are tangents to a circle with centre O. Find the unknown angles giving reasons.**

Cyclic Quadrilateral

- **Quadrilateral is a four sided plane shape**
- **A cyclic quadrilateral is a quadrilateral that is enclosed in a circle such that the four vertices touch the circumference of the circle.**

Note: the four points where the vertices touch are referred to as concyclic points.

P **Q**

S **R**

Theorem:

The opposite angles in a cyclic quadrilateral are supplementary.

Note: Two angles are supplementary if their sum is 180 and complementary if their sum is 90.

Given: A cyclic quadrilateral ABCD in a circle with centre O.

To prove: <BAD + < BCD = 180

Construction: Join OB, OD

B

A a

2c O 2a C

D

Proof: Using letters in the diagram, Let <BAD = a

Reflex BOD = 2a (angle at the centre is twice the angle at the circumference)

Let < BCD = c

Obtuse BOD = 2c (angle at the centre is twice the angle at the circumference)

But 2a + 2c = 360 (angle at a point)

⇒ 2(a + c) = 360

⇒ a + c =

∴ a + c = 180^o

Theorem

The exterior angle of a cyclic quadrilateral is equal to the interior opposite angles. Using the letters in the diagram,

P **Q**

a_1 **b_1**

d_1 **c_1** **a_2** **T**

S **b_2**

U

Given: A Cyclic quadrilateral PQRS

To prove:

Construction: Produce SR to T and PS to U.

Proof:

(opposite angles of a cyclic quadrilateral)

(angles on a straight line)

⇒

Similarly;

(opposite angles of a cyclic quadrilateral)

(angles on a straight line)

⇒

Class Activity

- **Find the lettered angles in each of the figures below;**

(a) **P**

a

M **b** **O** **N**

115^0

Q

Q

(b)

R **q**

15^0

S **42^0** **P**

T **U**

PRACTICE EXERCISE

- **O is the centre of the circle PQRST. If <SPT = 42^o, <PST = $55^o$and <PSQ = 15^o, Find <QRS.**

P

42^0

T **Q**

55^0 15^0

S R

- **Find angle h in the diagram below;**

65^0

75^0

h

- **In the diagram, O is the centre of the circle and PQRS is a cyclic quadrilateral. Find the value of .**

1. **25^0B. 65^0C. 115^0 D. 130^0 (SSCE 2008)**

-

(4) . In the diagram, P, Q, R, S are points on the circle, PQS = 30^0, PRS = 50^0 and PSQ = 20^0. What is the value of ?

1. **260^0 B. 130^0 C. 100^0 D.80^0 (SSCE 2006)**

(5) In the diagram, PQ is a diameter of the circle and <PRS = 58^0. Find <STQ.

1. **29^0 B. 32^0 C. 42^0 D. 53^0 (SSCE 2001)**

ASSIGNMENT

1. **In the diagram, PQR is a circle with centre O. <QRP = 50^0, <PQO = $30^0$and <ORP = m. Find m.**
2. **20^0 B. 25^0 C. 30^0 D. 50^0 (SSCE 1999)**

2. **In the diagram, PST is a tangent to circle VSU centre O. <SVU = $50^0$and UV is a diameter. Calculate <RSV.**
3. **90^0 B. 50^0 C. 45^0 D. 40^0 (SSCE 1999)**

3. **In the diagram below, O is the centre of the circle and. If ∠find the ∠BAO.**
4. **(SSCE 1995)**

4. **In the diagram, PQ is the tangent to the circle RST at T. /ST/ = /SR/ and <RTQ = 68^0. Find <PST.**

5. **68^0 B. 62^0 C. 61^0 D. 56^0 E. 34^0 (SSCE 1994)**

5. **The diagram shows a circle PQRS in which <PQR = $54^0$and <SPQ = 97^0. Find <PQS.**
6. **61^0 B. 51^0 C. 43^0 D. 39^0 E. 29^0 (SSCE 1994)**

WEEK 2

TOPIC: CIRCLE THEOREM

- **The angle which an arc subtends at the centre is twice the angle it subtends at the circumference.**
- **Angles in the same segment of a circle are equal.**
- **Angle in a semi-circle.**
- **Tangent to a circle.**

PROOF OF (i) ***The angle which an arc subtends at the centre is twice the angle it subtends at the circumference.***

The angle which an arc (or a chord) of a circle subtends at the centre of the circle is twice the angle which it subtends at any point on the remaining part of the circumference.

Note: An arc of a circle is any connected part of the circle's circumference.

A chord which is not a diameter divides the circle into two arcs- a major and a minor arc.

Given: An arc AB of a circle with 'O' and a point 'P' on the circumference.

To Prove: A

Construction: Join and produce the line to a point D

Sketch:

P P

X_1 y_1 A x_2 X_1 y_1

o o y_2 B

X_2 y_2

A D B D

-

(i) (ii)

-

P

O X_1 y_1

D

(iii) X_2 y_2

A B

Proof: since (radii in the same circle)

(base angles of isosceles AP)

AD = (exterior angle of AP)

AD = 2 (since)

Similarly, BOD = 2

In (a) acute/obtuse AOB = AOD + BOD

In (b) reflex AOB = AOD + BOD

= 2 + 2

= 2()

= 2APB

In (c) AOB = AOD – BOD

= 2

= 2()

= 2APB

AOB = 2APB (in all cases)

(2) in the diagram below, O is the centre of the circle ACB. If <CAO = 26^o and <AOB = 130^o, calculate (a) <OBC and (b) <COB (WAEC)

C

α α

O

26^0 130^0

A B

Solution:

ACB =

= 65^o

= α + α

α =

= 32.5^o

AOC = 180 – (26 + 32.5)

= 180 – 58.5

= 121.5^o

COB = 360 – (130 + 121.5) (angle at a point)

= 360 – 251.5

= 108.5^o

∴ OBC = 180 – (108.5 + 32.5)

= 180 – 141

= 39^o

(3) Given a circle with centre O while A,B and C are points on the circumference. Find <ABC, if the obtuse <AOC = 125^o

B

A **C**

125^0

O

Solution:

Reflex AOC = 360 – 125 (angle at a point)

= 235^o

∴ ABC = (angle at the centre is twice the angle at the circumference)

= 117.5^o

Class Activity

1. **Find the lettered angles in each of the figures below;**

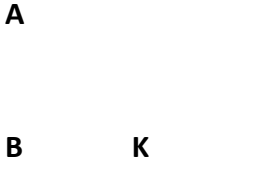
(a) 

(b) 

2. **In the diagram, ABCD is a circle centre O. AC and BD intersect at right angles at K. Angle COD is 130^o, calculate angles (i) DAC**

(ii) ADB

(iii) AOB (WAEC)



O

130^0 D

C

3. **(a) Prove that the angle which an arc of a circle subtends at the centre is twice that which it subtends at any point on the remaining part of the circumference.**

- **In the diagram below, O is the centre of the circle <OQR = $32^o$and <MPQ = 15^o**

Calculate: (i) <QPR

(ii) <MQO (WAEC)

P

15^0

M S

O

Q 32^0 R

PROOF OF :Angles in the same segment of a circle are equal.

Given: points A,B and C on the major segment of a circle ABCDE with centre O.

To Prove: <EAD = <EBD = <ECD

Construction: Join EO; DO

B

A **C**

P **q** **r**

O

E **D**

Proof:

EOD = 2p (angle at the centre is twice angle at the circumference)

EOD = 2q (angle at the centre is twice angle at the circumference)

EOD = 2r (angle at the centre is twice angle at the circumference)

⇒ p = q = r

∴ EAD = EBD = ECD

(2) The diagram below shows a circle ABCD in which <DAC = 55^o and <BCD = 100^o, find BDC.

A **B**

55^0

D **100^0** **C**

Solution:

<CAD = <CBD = 55^o (angles on the same segment)

∴ <BDC + <CBD + <BCD = 180^o (sum of the angles of a triangle)

⇒ <BDC + 55^o + 100^o = 180^o

⇒ <BDC = 25^o

(3) In the diagram below, PQRS is a circle if /PT/ = /QT/ and <QPT = 70^o, calculate < PRS? (WAEC)

P **Q**

70^0

S **R**

In PQT, PT = TQ (isosceles triangle)

∴ QPT = PQT = 70^o

But PQ = SR common chord

SRT = QPT = 70^o (alternate angle)

Class Activity

1. **Find the lettered angles in each of the figures below;**

- **(b)**

M e I h f

O d

N 15 50 40 g

55^0

PROOF OF:Angle in a semi-circle

Given: PQ is the diameter of a circle with c entre O and R is any point on the circumference.

To Prove: PRQ = 90^o

Construction: PR, RQ

R

P Q

Proof:

<POQ = 2PRQ (angle at the centre is twice that at the circumference)

But POQ = 180^o (angle on a straight line)

$\therefore$ 2PRQ = 180^o

PRQ =

$\therefore$ PRQ = 90^o

(2) In the diagram, O is the centre of the circle. If <BAC = 55^o, find the value of <ACB

B

A C

Solution:

<ABC = 90^o (angle in a semi-circle)

<ABC + <ACB + <BAC = 180^o (sum of angles in a triangle)

$\Rightarrow 90^o$ + <ACB + $55^o = 180^o$

<ACB + $145^o = 180^o$

<ACB = 180 – 145

< ACB = 35^o

(3) Find the values of the lettered angles in the figure below;

B

X 60

y

A D O C

Solution:

<ABC = 90

∴ X = 90 – 60

= 30

In ABD, <ADB = 90 (perpendicular bisector)

∴ x + y + 90 = 180

30 + y + 90 = 180

y = 180 – 120

y = 60^{o}

Class Activity

- **Find the values of the lettered angles in the figures below;**
- **54**

64 a

O

-

30

y O x

Tangent to a circle

The tangent to a circle is a straight line drawn to touch the circle at a point. The point where the line touches the circle is referred to as the point of contact.

A secant is a straight line that cuts a given circle into two clear points

Point of contact

Note:

- **A tangent to a circle is perpendicular to the radius drawn to its point of contact.**
- **The perpendicular to a tangent at its point of contact passes through the centre of the circle.**

Theorem:

Two tangents drawn to a circle from an external point are equal in length.

Given: An exterior point T of a circle with centre O. TY and TX are tangents to the circle at X and Y.

X

O

T

Y

To Prove: /TX/ = /TY/

Construction: Join TX, TO and TY

Proof: In triangles TXO and TYO

TXO = TYO = 90 (tangent perpendicular to radius)

/XO/ = /YO/ (radius)

/TO/ = /TO/ (common)

∴ /TX/ = /TY/

- **AB and AC are tangents from a point A to a circle centre O. If <BAC = 54^o, find the value of X**

O X

54^0

Solution:

ABO = ACO (tangents to a circle from an external point are equal)

ABO = ACO = 90 (tangents perpendicular to radius)

∴ ABO + ACO + BAC + X = 360 (sum of angles in a quadrilateral)

⇒ 90 + 90 + 54 + X = 360

⇒ 234 + X = 360

⇒ X = 360 – 234

∴ X = 126^o

- **Calculate PRQ**

R

P

O

88^0

Q **T**

Solution:

PTQ = 88^o

Join PO and QO

OP and OQ are radii

TQO = TPO = 90 (radii perpendicular to tangent)

∴ OPT + OQT = 180

PTQ + QTP = 180

QOP = 180 – 88

= 92^o

But QRP = ½ (QOP) (angle at centre is twice angle at the circumference)

= ½ (92)

= 46

∴ PRQ = 46^o

Class Activity

1. **Calculate the values of the marked angles below;**

•

O **28**

x R

(b)

O

45 45

PRACTICE EXERCISE

- **PQRT is a circle. /ST/ = /RS/ and TSR = 51^{o}, find POR (JAMB)**

S

O T

P R

Q

- **AB and CB are tangents to the circle. Given that CBA = 54^o, calculate <ADC**

(NECO)

A

D **54^o** **B**

C

- **TP is a tangent to the circle TRQ with centre O. if <TPO = 28^{o}and <ORQ = 15^o. Find (a) <RQT (b) <QTO (NECO)**

T

O

15 **28**

R **P**

Q

- **PQRST lie on the circumference of the circle with centre O. The chords PS and RT intersect at V and the chords PT and RS produced meet at X as shown below;**

T

P **X**

O V

Q

S

R

Given that the obtuse POR = 4 PXR

Prove that: (a) SVT = 3 PXR , (b) PSR = PQR (London G.C.E)

- **O is the centre of the circle. <OQR = 32^{o}and <TPQ = 15^{o}, Calculate (a) <QPR (b) <TQO (WAEC)**

P

15^{0}

T **S**

O

32^0

Q R

ASSIGNMENT

- **Find the values of the lettered angles in the figure below;**

63^0

c

O

- **In the diagram, AB is the diameter. <ABC = $(5x + 3)^o$and <BAC = $(5y + 7)^o$. Express y in terms of x**

C

5y+7 5x+3

A B

- **The diagram below is a circle with its centre at O. Find the value of (a)**

3x+3

y-8

60

- **P,Q,R and S are points on the circle. If <PSQ = 30^o, <PRS = $50^o$and <PSQ = 20^o, what is the value of**

P

Y x Q

30

20^0

S 50 R

- **Calculate the values of the marked angles below;**

18

30 y

1. **TS is a tangent to a circle PQRS. If /PR/ = /PS/ and PQR = 117^o, calculate RST (WAEC)**

Q

R P

T S

WEEK 3 TOPIC: TRIGONOMETRY (Sine and Cosine Rule)

CONTENT:

- **Derivation and application of sine rule.**
- **Derivation and application of cosine rule.**

SINE RULE

Given any triangle ABC (acute or obtuse), with the angles labelled with capital letters A, B, C and the sides opposite these angles labelled with the corresponding small letters a, b, and c respectively as shown below.

C

C

b a b a

A c B A c B

The sine rule states that;

OR

<u>PROOF OF THE RULE</u>

Using Acute – angled triangle

C

b h a

A c B

Given: **Any ΔABC with B acute.**

To prove: $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$

Construction: **Draw the perpendicular**

from C to AB.

Proof: ***Using the lettering in the diagram above.***

sinA = $\frac{h}{b}$

h = bsinA ---------- (1)

sinB = $\frac{h}{a}$

h = asinB ---------- (2)

From equation (1) and (2)

bsinA = asinB

**** $\frac{a}{sinA} = \frac{b}{sinB}$

Similarly, by drawing a perpendicular from B to AC

$\frac{a}{sinA} = \frac{c}{sinC}$

Q.E.D

Using Obtuse – angled triangle

C

b

a h

A c B

Given: **any ΔABC with B obtuse**

To Prove: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Construction: **Draw the perpendicular**

from C to AB produced.

Proof: ***With the lettering in the diagram.***

$\sin A = \frac{h}{b}$

$h = b\sin A$ ----------(1)

$\sin(180 - B) = \frac{h}{a}$ but $\sin(180-q) = \sin q$

$\therefore \sin B = \frac{h}{a}$

$h = a\sin B$ ----------(2)

From equation (1) and (2)

bsinA = asinB

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Similarly, by drawing a perpendicular from A to CB produced.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Q.E.D

APPLICATION OF SINE RULE

The sine rule is used for solving problems of triangle, which are NOT right – angled, and in which either two sides and the angle opposite one of them are given or two angles and any side are given.

Example 1:

In DABC, a = 9cm, B = 110^0, b = 13cm. Solve the triangle completely.

Solution:

The diagram representing the information above is given below as

C

b = 13cm a = 9cm

110^0

A c B

Using sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{9}{} = \frac{13}{}$$

sinA sin110^0

9sin 110^0 = 13sinA

sinA = $\frac{9\sin 70^0}{13}$

sinA = 0.6506

A = sin^{-1} 0.6506

A = 40.6^0

\ A » 41^0 (*nearest degree*)

To find angle C

A + B + C = 180^0 *[sum of <s in a D]*

41^0 + 110^0 + C = 180^0

C = 180^0 - 151^0

\ C = 29^0

To find side c, use sine rule

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{9}{\sin 41} = \frac{c}{\sin 29}$$

c = $\frac{9\sin 29}{\sin 41}$

c = 6.65cm

\ c = 6.7cm

Example 2:

In DPQR, given that P = 50^0, Q = 60^0,

r = 7.5cm. Find (i) p (ii) q

Solution:

R

q p

50^0 60^0

P 7.5cm Q

(i) P + Q + R = 180^0 ***[sum of <s in a Δ]***

$50^0 + 60^0 + R = 180^0$

$R = 180^0 - 110^0$

$R = 70^0$

Using sine rule

$\frac{r}{\sin R} = \frac{p}{\sin P}$

$\frac{7.5}{\sin 70^0} = \frac{p}{\sin 50^0}$

$p = \frac{7.5\sin 50^0}{\sin 70^0}$

p = 6.11cm

\ p » 6cm

(ii) ***Using sine rule***

$\frac{r}{\sin R} = \frac{q}{\sin Q}$

$\frac{7.5}{\sin 70^0} = \frac{q}{\sin 60^0}$

$q = \frac{7.5 \sin 60^0}{\sin 70^0}$

q = 6.9cm

\ q » 7cm

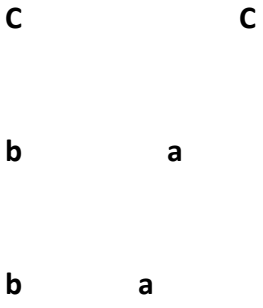
Class Activity:

Find the missing sides and angles of the following triangles. Calculate all angles to the nearest degree and all sides to 1 decimal place.

- **DABC, given that $B = 68^0$, b = 27m and a = 22m.**
- **DPQR, given that $Q = 121^0$, q = 57km and r = 17km.**
- **DABC, given that $C = 27^0$, c = 7cm and b = 13cm.**

COSINE RULE

Given any triangle ABC (acute or obtuse), with the angles labeled with the capital letters A, B, C and the sides opposite these angles labeled with the corresponding small letters a, b, and c respectively as shown below



A c B B **A c**

The cosine rule states that

$a^2 = b^2 + c^2 – 2bc \cos A$

$b^2 = a^2 + c^2 – 2ac \cos B$

$c^2 = a^2 + b^2 – 2ab \cos C$

PROOF OF THE RULE

Using acute – angled triangle

C

b a

h

A c- x D x B

c

Given: **Any DABC with B acute.**

To prove: **$b^2 = a^2 + c^2 – 2ac \cos B$**

Construction: **Draw a perpendicular**

from C to AB.

Proof: ***With the lettering in the diagram.***

$b^2 = (c - x)^2 + h^2$ ***(Pythagoras)***

$= c^2 – 2cx + x^2 + h^2$

But in D BCD, $a^2 = x^2 + h^2$

\ $b^2 = c^2 – 2cx + a^2$ ----------(1)

In DBCD,

cosB = x

a

\ x = a cos B

From Eqn (1)

$b^2 = c^2 + a^2 – 2cx$

$b^2 = c^2 + a^2 – 2ca \cos B$

Q.E.D

Using obtuse – angled triangle

C

b

a h

A c B x D

c + x

Given: **Any DABC with B obtuse**

To prove: **$b^2 = a^2 + c^2 – 2ac \cos B$**

Construction: **Draw the perpendicular**

from C to AB produced.

Proof: ***With the lettering in the diagram.***

$b^2 = (c + x)^2 + h^2$

$= c^2 + 2cx + x^2 + h^2$

But in DBCD

$a^2 = x^2 + h^2$ *(by Pythagoras)*

\ $b^2 = c^2 + 2cx + a^2$

ie $b^2 = a^2 + c^2 + 2cx$ -------- (1)

In DBCD, cosB = $\frac{x}{a}$

cos (180 – B) = $\frac{x}{a}$

-cosB = $\frac{x}{a}$

\ x = -acosB

From Eqn (1)

$b^2 = a^2 + c^2 + 2c(-a\cos B)$

\ $b^2 = a^2 + c^2 – 2ac\cos B$

Q.E.D

Similarly, $a^2 = b^2 + c^2 – 2bc\cos A$

$c^2 = a^2 + b^2 – 2ab\cos C$

APPLICATIONS OF COSINE RULE

Cosine rule can be used for solving problems involving triangles, which are not right–angled, in which two sides and the angle between the two sides are given i.e. two sides and the included angle.

Secondly, the formula can be used to find the angles of a triangle when the three sides of the triangle are given.

USING COSINE RULE TO FIND THE MISSING SIDE OF A TRIANGLE

Examples:

- ***In DABC, given that A = 65^0, b = 9cm and c = 12cm, Find a.***

Solution:

C

9cm a

65^0

A 12cm B

Using cosine rule

$a^2 = b^2 + c^2 – 2bccosA$

$= 9^2 + 12^2 – 2x9x12cos65$

= 81 + 144 – 216cos65

= 225 – 216 x 0.4226

= 225 – 91.28

= 133.72

a = Ö133.72

\ a = 11.56cm.

- ***Find the value of q in the figure below.***

R

q

5m

112^0

P 7m Q

Solution:

Using cosine rule

$q^2 = p^2 + r^2 – 2prcosQ$

$= 5^2 + 7^2 – 2x5x7cos112^0$

= 25 + 49 – 70[-cos(180 – 112)]

= 74 – 70(-cos 68)

= 74 + 70cos68

= 74 + 70 x 0.3746

= 74 + 26.222

= 100.222

q = Ö100.22

\ q = 10.01

\ q » 10m

-

-

- ***In DABC, B = 130^0, a = 4.62cm and c = 6.21cm, Calculate b.***

-

Solution:

A

b

6.21cm

130^0

B 4.26cm C

Using cosine rule

$b^2 = a^2 + c^2 – 2ac \cos B$

$= 4.62^2+6.21^2–2x4.62x6.21\cos130^0$

= 21.34+38.56–57.38[-cos180–130]

= 59.9 - 57.38 [-cos 50]

= 59.9 + 57.38 x 0.6428

= 59.9 + 36.88

= 96.78

b^2 = Ö96.78

\ b = 9.8cm.

Class Activity:

Solve the following questions and approximate all answers to 1 decimal place.

(1) In DABC, B = 53^0, c = 45km and

a = 63km. Find b.

(2) In DPQR, Q = 111^0, r = 47km and p =

39km. Find q.

(3) In DABC, B = 87^0, a = 25m and c =

19m. Find b.

(4) In DABC, B = 142^0, a = 33km and c =

27km. Find b.

USING COSINE RULE TO CALCULATE ANGLES

Cosine rule can also be used to calculate the angles of a triangle when the three sides are given. This is done by making the cosine of the desired angle the subject of the formula.

E.g. If

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$2bc\cos A = b^2 + c^2 - a^2$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Similarly, $$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

and $$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

This formula is used to calculate the angles of a triangle when all the three sides of the triangle are given.

Examples:

Find the angles of the D ABC given that a = 7cm, b = 6cm and c = 5cm.

Solution:

C

6cm 7cm

A 5cm B

To find angle A,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{6^2 + 5^2 - 7^2}{2 \times 6 \times 5}$$

$$= \frac{36 + 25 - 49}{60}$$

cosA = 0.2000

$$A = \cos^{-1} 0.2000$$

$$\therefore A = 78.5^0 \quad \text{---------- (1)}$$

To find angle B,

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{7^2 + 5^2 - 6^2}{2 \times 7 \times 5}$$

$$= \frac{49 + 25 - 36}{70}$$

cosB = 0.5429

$$B = \cos^{-1} 0.5429$$

$$\therefore B = 57.1^0 \quad \text{----------(2)}$$

To find angle C,

$$\cos C = \frac{7^2 + 6^2 - 5^2}{2 \times 7 \times 5}$$

$$= \frac{49 + 36 - 25}{84}$$

cosC = 0.7143

$C = \cos^{-1} 0.7143$

$\backslash C = 44.4^0$ ---------- (3)

Check: **From Eqn (1), (2) and (3).**

$A + B + C = 78.5^0 + 57.1^0 + 44.4^0$

$= 180^0$

Class Activity

Using cosine rule, calculate the three angles of the following triangles whose sides are given below. Approximate all your answer to the nearest degree.

(1) D XYZ, x = 10m, y = 16m and

z = 13m.

(2) D PQR, p = 25km, q = 30km, and

r = 8km.

(3) DABC, a = 5.7cm, b = 3.5cm and

c = 4.3cm.

GENERAL PROBLEM SOLVING USING SINE AND COSINE RULE.

A combination of sine and cosine rule can be used to solve a given problem, as we shall see subsequently.

Example 8:

Find the value of the following from the diagram below (i) x (ii) q (iii) ôBDô.

C

13cm

D 43^0

xcm 7cm

q 35^0 125^0

A B

Solution:

(i) *Using sine rule*

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{7}{\sin 35^0} = \frac{x}{\sin 125^0}$$

$$X = \frac{7 \sin 125^0}{\sin 35^0}$$

$$X = \frac{7 \sin 55^0}{\sin 35^0}$$

x = 9.99cm

\ x » 10cm

(ii) *Using sine rule*

$$\frac{10}{\sin 43^0} = \frac{13}{\sin q}$$

$$10 \sin q = 13 \sin 43^0$$

$$\sin q = \frac{13 \sin 43^0}{10}$$

$$\sin q = 0.8866$$

$$q = \sin^{-1} 0.8866$$

$$q = 62^0$$

(iii) To find /BD/

D 13cm C

7cm

B

BCD = BCA + ACD ---------- (1)

BCA = 180^0 – (125^0 + 35^0) *(sum of Ds in DABC)*

= 180^0 - 160^0

= 20^0

ACD = 180 – (43^0 + q^0)

= 180 – (43^0 + 62^0)

= 180 - 105^0

= 75^0

From (1)

BCD = 20^0 + 75^0

= 95^0

Using cosine rule to find /BD/

/BD/^{2} = b^2 + d^2 – 2bdcosC

= 13^2 + 7^2 – 2 x 13 x 7cos95^0

= 169 + 49 – 182[-cos180–95]

= 218 – 182 [-cos 85]

= 218 + 182 x 0.0872

= 218 + 15.87

/BD/^{2} = 233.87

/BD/ = Ö233.87

/BD/ = 15.29cm

/BD/ = 15.3cm *(1. d.p)*

Example 9:

Find the unknown sides and angles of a triangle ABC given that C = 69^0 , a = 9cm and b = 6cm. Give answer to 3 significant figure.

Solution:

Using cosine rule

$c^2 = a^2 + b^2 – 2abcos\ C$

$= 81 + 36 – 108\ cos69^0$

= 117 – 108 x 0.3584

= 118 – 38.71

= 79.29

C = Ö79.29

C = 8.90cm

To get angle B, we shall use sine rule

$$\frac{b}{sinB} = \frac{c}{sinC}$$

$$\frac{6}{sinB} = \frac{8.9}{sin69^0}$$

$6sin69^0 = 8.9sinB$

$$sinB = \frac{6sin69^0}{8.9}$$

sinB = 0.6294

$B = sin^{-1}\ 0.6294$

\ B = 39^0

To get angle A,

A + B + C = 180^0 *[sum of Ls in a D]*

A + 39^0 + 69^0 = 180^0

A = 180^0 - 108^0

\ A = 72^0

Class Activity:

(1) The figure below is a trapezium

ABCD, in which /AB/ is parallel to

/DC/, and the lengths of the sides are

as shown below.

108^0

Calculate the value of the following

(i) /AC/ (ii) ABC

(2) R

3cm 8.3cm

P

The figure above is a triangle PQR with the dimension as shown above. Calculate the following (i) RPQ (ii) /QS/

(3) In DPQR p:q:r = Ö3:1:1. Calculate the

ratio P:Q:R in its simplest form.

(WAEC).

(4) Calculate the angles of the triangles

whose sides are in the ratio 4:5:3.

(5) Given a triangle PQR, in which

/PQ/ = 13cm, /QR/ = 9cm,

/PR/ = 7cm and QR is produced to S

so that /RS/ = 6cm. Calculate the

following. (i) cos PRS (ii) /PS/

(6) Find the value of the following from

the diagram below.

(i) x

(ii) DAB

7.3cm xcm 6cm

PRACTICE EXERCISE

- **The angle of elevation of the top of a building measured from point A is 25°. At point D which is 15m closer to the building, the angle of elevation is 35°Calculate the height of the building.(Hint: use sine rule)**
- **The angle of elevation of the top of a column measured from point A, is 20°. The angle of elevation of the top of the statue is 25°. Find the height of the statue when the measurements are taken 50 m from its base(Hint: use sine rule)**
- **Find the values of the unknown sides and angles**

ASSIGNMENT

- **A fishing boat leaves a harbour (H) and travels due East for 40 miles to a marker buoy (B). At B the boat turns left onto a bearing of 035°and sails to a lighthouse (L) 24 miles away. It then returns to harbour.**

1. **Make a sketch of the journey**

2. **Find the total distance travelled by the boat. (nearest mile)**

- **A fishing boat leaves a harbour (H) and travels due East for 40 miles to a marker buoy (B). At B the boat turns left and sails for 24 miles to a lighthouse (L). It then returns to harbour, a distance of 57 miles.**

1. **Make a sketch of the journey.**
2. **Find the bearing of the lighthouse from the harbour. (nearest degree)**

- **Find the unknown sides and angles**

WEEK 4

TOPIC: TRANSFORMATIONS

CONTENT:

(a) Translation of points and shapes on the Cartesian plane.

(b) Reflection of points and shapes on the Cartesian plane.

(c) Rotation of points and shapes on the Cartesian plane.

(d) Enlargement of points and shapes on the Cartesian plane.

When the position or dimensions (or both) of a shape changes, we say it is transformed. The image is the figure which results after transformation of the shape. If the image has the same dimension as the original shape, the transformation is called a congruency. (Two shapes are congruent if their corresponding dimensions are congruent). A transformation is a mapping between two shapes.

Translation of points and shapes on the Cartesian plane.

A Translation is a movement in a straight line. Under a translation every point in a line or plane shape moves the same distance in the same direction by a fixed translation or displacement vector. Note:

In general, if the position vector of a point is given by the translation the position vector of its image is . We write and say maps to

Every point in the shape moves the same distance in the same direction.

Examples:

1. **A translation maps (5, -4) on to (3, -6).**

- **What is the displacement vector?**
- **What is the image of (-2, 7) under this translation?**

Solution:

- **Let the displacement vector be ,**

Point + displacement = image

- **,**

Hence, Image under this translation is

Class Activity:

1. **What is the image of P(-2, -5) under the translation**

2. **The vertices of triangle ABC are represented by the coordinates A(-2,-1), B(2,0), C(2,-2). Draw this triangle on graph paper and show its image under the translation**

Reflection of points and shapes on the Cartesian plane

A reflection is the image you see when you look in a mirror. The line of the mirror is a line of symmetry between the object shape and its image. In a Cartesian plane, there are infinitely many lines of reflection. The following describes some of the important ones.

Reflection in the x-axis:

The point P(4,2) is reflected in the x-axis. Its image P'(4,-2) is the same distance from the x-axis as the point P. if the position vector of a point is , the position vector of its image under reflection in the x-axis is . This gives the mapping

Reflection in the y-axis:

If a point is reflected in the y-axis, its image P'(-2,1) is the same distance from the y-axis. If the position vector of a point is , the position vector of its image under reflection in the y-axis is . This gives the mapping

Reflection in the line y = x:

The image of the vector P(2,5) is P'(5,2) after reflection in the line y = x, this mapping is equivalent to

Reflection in the line y = -x:

The image of P(1,3) is P'(-3,-1) after reflection in the line y = -x. This mapping is equivalent to

Example:

1. **If a point P has the coordinates (5,-2), find its reflection in the;**

(a) x-axis

(b) y-axis

(c) line y = x

(d) line y = -x

Solution:

Let the image of P be P' after reflection.

- **In the x-axis, , the coordinate of P', the image of P, are (5,2)**
- **In the y-axis, , the coordinate of P', the image of P, are (-5, -2)**
- **In the line y = x, , the coordinate of P', the image of P are (-2,5)**
- **In the line y = -x, , the coordinate of P', the image of P are (2, -5)**

Class Activity:

1. **State the coordinate of the image of point A(3,2) after reflection in the;**

(a) x-axis

(b) y-axis

(c) line y = x

(d) line y = -x

2. **The coordinates of triangle ABC are A(1,6), B(4,6), C(2,5). Find the coordinate of the image of triangle ABC after reflection in (a) the line y = x (b) the line y = -x**

Rotation of points and shapes on the Cartesian plane.

If a point P, whose position vector is , is rotated through in the anticlockwise sense about the origin, by construction, the position vector of the image P' is;

- **for**
- **for**
- **for**

If the rotation is clockwise, the position vector of the image, P' is;

- **for**
- **for**
- **for**

Examples:

1. **If the point P(2,4) is rotated anticlockwise through 90^0about the origin, determine the coordinates of the image.**

Solution:

Under rotation through 90^0 anticlockwise; ,

Therefore,, the coordinate of the image are (4,2)

2. **The point T() is rotated anticlockwise through a half turn (that is 180^0) about the origin. Determine the coordinates of the image.**

Solution:

Under rotation through 180^0 anticlockwise; ,

Therefore, , the coordinate of the image are (3,)

Class Activity:

1. **Determine the coordinates of the image of the point P(-4,3) if it is rotated anticlockwise through 90^0about the origin.**

2. **The point B(6, -2) is rotated through a half turn about the origin. Find R(B), the image of B under rotation if**

- **the rotation is clockwise**
- **the rotation is anticlockwise**

Enlargement of points and shapes on the Cartesian plane.

An enlargement is a transformation in which a shape is made bigger or smaller according to a given scale factor and a centre of enlargement which does not change.

PRACTICE EXERCISE:

1. **T is a translation which moves the origin to the point (3,2). R is a anticlockwise rotation of 90^0about the origin. A is the point (2,-5), B is (-1,4) and C is (-4,4). Find the coordinates of the image of:**

- **A after translation T**
- **B after rotation R**
- **C if it is first translated by T and then rotated by R.**

2. **A'(5,5), B'(-5,10), C'(0,20) are the images of A(2,2), B(-2,4), C(0,8) after a transformation F.**

- **Using a scale of 1cm to 2units, draw the triangles ABC and A'B'C' on the same Cartesian plane.**
- **Describe fully the transformation F**
- **Find the coordinates of the image of triangle ABC after rotation 270^0clockwise about the point (3,2)**

3. **Triangle A(0,2), B(1,0), C(2,1) is first enlarged about point (1,-2) with scale factor 2. It is then reflected in the line x = -1. Find the vertices of its final image.**

4. **Quadrilateral Q is rotated through 180^0about the point (0,2). The result is then enlarged by a scale factor of -2 with the origin as centre. Find the coordinates of the vertices of the final image of Q.**

5. **(a) Using a scale of 1cm to represent 1 unit on each axis, draw x and y-axes for Draw a triangle with vertices (-1,1), (-1,10), (-4,7) and label it F.**

(b) A transformation R maps triangle F on to the triangle R(F) which has vertices (0,-2), (9,-2), (6,1). Draw triangle R(F) and fully describe the transformation R.

(c) M is a reflection in the line y = x. Find by drawing, the coordinates of the vertices of the triangle M(F).

WEEK 5

BEARINGS

This is a system of measuring the location of points on the earth's surface in relation to another using the four cardinal points of the earth. i.e. the North, South, East and West.

There are two major ways of measuring the bearings of points. They are

(i) The three-digit bearing (True bearing).

(ii) The points of compass bearing.

The Three-digit bearing or True bearing

This type of bearing is normally expressed using three digits as the name implies e.g. 003^0, 007^0, 025^0, 067^0, 125^0, 218^0 e.t.c.

The bearing is normally read from the North Pole in a clockwise direction until the desired point is reached.

Example 1:

The bearing of B from A is 075^0, what is the bearing of A from B?

Solution: **N**

B 90^0

75^0 90^0

N 075^0

A

The bearing of A from B is

$90^0 + 90^0 + 75^0 = 255^0$

(This is read from the North Pole at point B)

Example 2:

The bearing of Y from X is 240^0, what is the bearing of X from Y?

<u>Solution:</u>

N

x 180^0

60^0

N 60^0

Y

The bearing of X from Y is 060^0

(This is read from the North Pole at point Y)

***<u>Example 3</u>*:**

The bearing of Q from P is 188^0, what is the bearing of P from Q?

-

-

<u>Solution:</u> **N**

P 180^0

8^0

N

Q 8^0

The bearing of P from Q is 008^0

(This is read from the North Pole at Q)

The Points of Compass Bearing

This type of bearing is usually read either from the North or South to any of the directions specified, East or West. It is usually started with the letters N or S denoting North or South and it is normally ended with the letters E or W denoting East or West i.e. Nq^0W, Nq^0E, Sq^0W, Sq^0E where q lie between 0 and 90^0 ($0^0 < q < 90^0$).

The first letter N or S as the case may be, signifies the point we are reading from and the last letters E or W signifies the direction we are reading to.

e.g.

$N65^0E$ Þ We are reading from the

North 65^0 towards the East.

$S30^0W$ Þ We are reading from the

South 30^0 towards the West.

$S17^0E$ Þ We are reading from the

South 17^0 towards the East.

We shall reframe the three examples under the three-digit bearing using point of compass bearing specifications.

Examples

- ***The bearing of B from A is $N75^0E$, what is the bearing of A from B?***

Solution:

-

-

-

N B

W E

75^0

S

N

A 75^0

W E

S

The bearing of A from B is $S75^0W$.

-

- ***The bearing of Q from P is $S8^0W$, what is the bearing of P from Q?***

N P

W E

8^0S

Q N 8^0

W E

S

The bearing of P from Q is $N8^0E$.

-

- ***The bearing of Y from X is $S60^0W$, what is the bearing of X from Y?***

<u>Solution:</u>

N X

W E

60^0

S N 60^0 Y

W E

S

The bearing of X from Y is N60^{0}E

NOTE THAT:

The bearing of a place is said to be due North if it is directly to the North; due South if it is directly down South; due East if it is directly towards the East and due West if it is directly towards the West.

-

- **B is due North of A**

B

N

W A E

S

- **B is due East of A**

N

A E B

S

- **B is due West of A**

N

A

W E

B S

- **B is due South of A**

N

A

W E

S

B

- **C is North East of B**

C

N

B E

- **B is directly South West of A**

N

A

W E

S

B

Class Activity

(1) What's the bearing of Q from P to the nearest whole degree?

1. **16^0 B. 17^0 C. 73^0 D.106^0 E. 164^0 (SSCE 1988)**

(2) Points X and Y are respectively 20km north and 9km east of a point O. What is the bearing of Y from X correct to the nearest degree?

1. **024^0 B. 114^0 C. 154^0 D. 204^0 E. 336^0 (SSCE 1989)**

(3) Town P is on a bearing 315^0 from town Q while town R is south of town P and west of town Q. if town R is 60km away from Q, how far is R from P?

1. **30km B. 42km C. 45km D. 60km E. 120km (SSCE 1992)**

(4) Points X and Y are respectively 12m North and East of point Z. Calculate /XY/.

1. **7m B. 12m C. 13m D. 17m E. 18m (SSCE 1992)**

(5) A plane flies 90km on a bearing 030^0 and then flies 150km due east. How far east of the starting point is the plane?

1. **120km B. 165km C. 195km D. $(150 + 45\sqrt{3})$ km E. 240km (SSCE 1993)**

PRACTICAL PROBLEMS ON BEARING.

THREE POINTS MOVEMENT WITH DISTANCE GIVEN

Examples:

(1) A dragonfly flew from point A to point B, 25m away on a bearing of 067^0. It then flew from point B to point C 17m away on a bearing of 143^0.

(a) How far is the dragonfly from the

starting point to the nearest metre?

(b) What is the bearing of the starting point from the dragonfly?

Solution:

We shall represent the movement of the dragonfly with a diagram.

N

B

143^0

q_1 q_2

25m 104^0 17m

q_3

C

N 067^0

b

A

(a)

Using cosine rule

$b^2 = a^2 + c^2 – 2ac \cos B$

$= 17^2 + 25^2 – 2x17x25 \cos 104^0$

$= 289 + 625 – 850 (-\cos 76^0)$

= 914 + 850 x 0.2419

= 914 + 205.6

= 1119.6

b = Ö1119.6

b = 33.46

\ b = 33m *(nearest metre)*

\ The dragonfly is approximately 33m from the starting point.

(b) Using sine rule

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{33.46}{\sin 104} = \frac{25}{\sin C}$

$33.46 \sin C = 25 \sin 104^0$

$\sin C = \frac{25 \sin 104}{33.46}$

$\sin C = \frac{25 \sin 76^0}{33.46}$

$\sin C = 0.7249$

$C = \sin^{-1} 0.7249$

$C = 46.47^0$

The bearing of the starting point from the dragonfly is

$= 360 – (q_3 + C)$

$= 360 – (37^0 + 46.47)$

$= 360^0 – 83.47^0$

$= 276.5^0$

$» 277^0$

(2) A ship in an open sea sailed from a point A to another point B, 15km away on a bearing of 310^0. It then sailed from the point B to another point C, 23km away on a bearing of 062^0.

- ***How far is the ship from the starting point?***
- ***What is the bearing of the starting point from the ship?***

-

Solution: ***(i)*** **q_4 C**

q_3

23km

062^0 q_1

B q_2 $_{68}{}^0$ b

15km 50^0N

q_5

A 310^0

Using cosine Rule

$b^2 = a^2 + c^2 – 2ac \cos B$

$= 23^2 + 15^2 – 2x23x15 \cos 68^0$

= 529 + 225 – 690 x 0.3746

= 754 – 258.474

b^2 = 495.526

b = Ö495.526

b = 22.3km

(ii) Using sine Rule

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{22.3}{\sin 68^0} = \frac{15}{\sin C}$$

$22.3 \sin C = 15 \sin 68^0$

$$\sin C = \frac{15 \sin 68^0}{22.3}$$

sin C = 0.6237

C = $\sin^{-1}$ 0.6237

C = 38.6^0

The bearing of the starting point from the ship is obtained from $360^0 – (q_3 + q_4 + C)$.

= $360^0 – (28^0 + 90^0 + 38.6^0)$

= $360^0 – 156.6^0$

= 203.4^0

\ The bearing of the starting point from

the ship is » 203^0

THREE POINTS MOVEMENT WITH SPEED AND TIME GIVEN

(Under this case, we shall be considering the bearing of ONE OBJECT moving to three different points with no distance given but the SPEED AND TIME OF THE VEHICLE GIVEN)

(3)A boat sails at 50km/h on a bearing of N52^{0}E for 1½ hours and then sails at 60km/h on a bearing of S40^{0}E for 2 hours.

- ***How far is the boat from the starting point?***
- ***What is the bearing of the starting point from the boat?***
- ***What is the bearing of the boat from the starting point?***

Solution: **N**

- **Q $q_1 40^0$**

75km 92^0

N 52^0 q_2 120km

P q_4 q q_3

R

Distance PQ = Speed x Time

= (50 x 1½) km

= (50 x $^3/_2$) km

= 75km

Distance QR = (60 x 2) km

= 120km

Using cosine rule

$q^2 = p^2 + r^2 – 2pr \cos Q$

$= 120^2 + 75^2 – 2x120x75 \cos 92^0$

$= 14400 + 5625 – 18000 [-\cos 180^0 - 92^0]$

= 20025 – 18000 (-cos 88)

= 20025 + 18000 x 0.0349

= 20025 + 628.2

= 20653.2

q = Ö20653.2

\ q = 143.7km.

(ii) Using sine rule

$\frac{q}{\sin Q} = \frac{r}{\sin R}$

$\frac{143.7}{\sin 92} = \frac{75}{\sin R}$

143.7 sinR = 75 sin 92^0

sinR = $\underline{75 \sin 92^0}$

143.7

sin R = $\underline{75 \sin 88^0}$

143.7

sin R = 0.5216

R = $\sin^{-1}$ 0.5216

R = 31.4^0

The bearing of the starting point from the boat is = N(R + q_3)^{0}W

= N(31.4^0 + 40^0)W

= N 71.4^0 W

» N 71^0 W

(iii)

The bearing of the boat from the starting point is read from the point P as S71^0E.

(4)An aircraft flew from an airport A to another airport B, on a bearing of 065^0 at an average speed of 300km/h for $2^1/_3$ hrs, It then flew from the airport B to another airport C, on a bearing of 320^0 at an average speed of 450km/h for 40min.

- ***How far is the aircraft from the starting point?***
- ***What is the bearing of the starting point from the aircraft?***
- ***What is the bearing of the aircraft from the starting point?***

-

Solution:(i) **q_3C**

q_4

300km

b **75^0 50^0**

q_2 **B**

65^0 q_5 **270^0**

N **q_1** **700km**

A

Distance = Speed x Time

Distance AB = (300 x $2^1/_3$) km

= (300 x $^7/_3$) km

= (100 x 7) km

= 700km.

Distance BC = (450 x 40) km

60

= (450 x $^2/_3$) km

= (150 x 2) km

= 300km.

Using cosine rule

$b^2 = a^2 + c^2 – 2ac \cos B$

$= 300^2 + 700^2 – 2x300x700 \cos 75^0$

$= 90000 + 490000 – 420000 \times 0.2588$

$= 580000 – 108696$

$= 471304$

b = Ö471304

\b = 686.5km

\ The aircraft is 686.5km from the starting point.

(ii) Using sine rule

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{686.5}{\sin 75} = \frac{700}{\sin C}$

686.5 sin C = 700 sin 75

sin C = $\frac{700 \sin 75}{686.5}$

sin C = 0.9849

C = $\sin^{-1}$ 0.9849

\ C = 80^0

The bearing of the starting point from the aircraft is read from point C.

i.e. = $q_3 + q_4 + C$

= $90^0 + 50^0 + 80^0$

= 220^0

(iii) **A + B + C = 180^0 *[sum of Ls in a D]***

$q_5 + 75^0 + 80^0 = 180^0$

$q_5 = 180^0 - 155^0$

$q_5 = 25^0$

The bearing of the aircraft from the starting point is = $90^0 - (q_5 + q_1)$

= $90^0 - (25^0 + 25^0)$

= $90^0 - 50^0$

= 040^0

(read from the point A)

Class Activity

(1) C

N

9m A

217^0

N 4m

B

320^0

From the diagram above, find the following

(i) ABC

(ii) /AC/

(iii)The bearing of A from C.

N

(2) P 122^0

21km

N

Q

200^0

15km

R

From the diagram above, find the following

(i) PQR

(ii) /PR/

(iii) The bearing of P from R.

(3) A town B is 12km from another town A on a bearing of 047^0 and another town C is 8km from town B on a bearing of 124^0.

(i) How far is town A from town C?

(ii) What is the bearing of town A from C?

(4) A ship sailing in an open sea moves from a point A on a bearing of 055^0 at a speed of 50km/h for 1½ hour to another point B. It then moves on a bearing of 143^0 at a speed of 40km/h for 2 hours to another point C.

(i) How far is the ship from the starting point?

(ii) What is the bearing of the ship from the starting point?

TWO DIRECTIONS WITH DISTANCE GIVEN

(Under this case, we shall be considering the bearing of TWO OBJECTS at different locations read from the same point or TWO OBJECT moving from the same point in two different directions AND the DISTANCES covered by the two objects GIVEN)

Examples:

(1) Two missiles A and B shot from the same point, Missile A was shot on a bearing of 058^0 and at a distance of 10km and missile B was shot on a bearing of 132^0 at a distance of 18km.

(i) How far apart are the missiles?

(ii) What is the bearing of missile A from missile B? ***(WAEC)***

Solution:

(i) **A**

10km

N 058^0

P q_1

132^0 42^0 74^0

p

18km

N

B

Using cosine rule

$P^2 = a^2 + b^2 – 2ab \cos P$

$= 18^2 + 10^2 – 2x18x10 \cos 74^0$

= 324 + 100 – 360 x 0.2756

= 424 – 99.216

= 324.784

P = Ö324.784

P = 18.0km

The two missiles are 18km apart.

(ii) Using sine rule

To find angle B,

$\frac{b}{} = \frac{p}{}$

sin B sin P

10 = 18

sin B sin 74^0

10 sin 74 = 18 sin B

sin B = 10 sin 74^0

18

sin B = 0.5340

B = $\sin^{-1}$ 0.5340

B = 32.3^0

To get the bearing of A from B

= $270^0 + q_2 + B$ *[q_2 = 42 (alternate <s)]*

= $270^0 + 42^0 + 32.3^0$

= 344.3^0

» 344^0

(2) Two points B and C are observed from a watch tower at point A. If B is 7km on a bearing of 063^0 and the other point C is 12km due south of A.

(i) How far apart are the two points?

(ii) What is the bearing of B from C?

- ***What is the bearing of C from B?***

Solution: **B**

7km q_3

063^0

A q_1

q_2 117^0

a

12km

C

(i) Using cosine rule

$a^2 = b^2 + c^2 – 2bc \cos A$

$= 12^2 + 7^2 – 2x12x7 \cos 117$

= 144 + 49 – 168 [-cos 180 – 117]

= 193 – 168 [-cos 63]

= 193 + 168 x 0.4540

= 193 + 76.27

$a^2 = 269.27$

a = Ö269.27

a = 16.4km

The two points are 16.4km apart.

(ii) Using sine rule

to find angle C

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{16.4}{\sin 117} = \frac{7}{\sin C}$$

16.4 sin C = 7 sin 117

$$\sin C = \frac{7 \sin 63}{16.4}$$

sin C = 0.3803

C = $\sin^{-1}$ 0.3803

\C = 22.4^0

» 22^0

\ The bearing B from C is 022^0

(iii) **The bearing of C from B is**

=180 + q_3

= 180 + 22^0

= 202^0

Class Activity

(1) Two men P and Q set off from a base camp R prospecting for oil. P move 20km on a bearing 205^0 and Q moves 15km on a bearing of 060^0.Calculate the

(a) Distance of Q from P

(b) Bearing of Q from P

(Give answers in each case correct to the nearest whole number).

SSCE, June 1996, No 12 (WAEC).

(2) Two boats A and B left a port C at the same time along different routes. B traveled a distance of 9km on a bearing of 135^0 and A traveled a distance of 5km on a bearing of 062^0.

(a) How far apart are the two ships?

(b) What is the bearing of ship B from A?

PRACTICE EXERCISE

(1) Two flying boats A and B left port P at the same time, A sailed on a bearing of 115^0 at an average speed of 8km/h and B sailed on a bearing of 241^0 at an average speed of 6km/h.

(a) How far apart are the flying boats after 1½ hour?

(b) What is the bearing of boat A from boat B?

(2) A man observed two boats P and Q at a sea sailing towards him at the point R. He observes P at a bearing of $N43^0W$ moving at an average speed of 20km/h and Q is on a bearing of $S52^0W$ moving at an average speed of 30km/h. If P took 2 hours to get to R and Q took 2½ hours to get to R.

(a) How far apart were the two boats when the man first noticed them?

(b) What was the bearing of P from Q?

(3)An aeroplane flew from city G to city H on a bearing of 150^0. The distance between G and H is 300km. It then flew a distance of 450km to city J on a bearing of 060^0. Calculate and correct to a reasonable degree of accuracy.

(a) The distance from G to J,

(b) How far north of H is J,

(c) How far west of H is G.

SSCE, Nov 1994, No 4 (WAEC).

(4) ***A girl moves from a point P on a bearing of 060^0 to a point Q, 40m away. She then moves from the point Q, on a bearing of 120^0 to a point R. The bearing of P from R is 255^0. Calculate, correct to three significant figures the distance between P and R.***

SSCE, Nov 1993, No 2b (WAEC).

ASSIGNMENT

(1) A man travels from a village X on a bearing of 060^0 to a village Y which is 20km away. From Y, he travels to a village Z, on a bearing of 195^0. if Z is directly east of X, calculate, correct to three significant figures, the distance of (i) Y from Z

(ii) Z from X.

SSCE, June 1995, No 10a (WAEC).

(2) A surveyor standing at a point X sights a pole Y due east of him and a tower Z of a building on a bearing of 046^0. After walking to a point W, a distance of 180m in the south-east direction, he observes the bearing of Z and Y to be 337^0 and 050^0 respectively.

(a) Calculate, correct to the nearest metre.

(i) /XY/

(ii) /ZW/

(b) if N is on XY such that XZ = ZN, find the bearing of Z from N.

SSCE, June 1998, No 10 (WAEC).

(3) An aeroplane flies from a town X on a bearing of $N45^0E$ to another town Y, a distance of 200km. It then changes course and flies to another town Z on a bearing of $S60^0E$. If Z is directly east of X, calculate correct to 3 significant figures.

(a) The distance from X to Z.

(b) the distance from Y to XZ.

(WAEC).

N

(3) A

210^0

N 50km

B 150^0

80km

C

(a) In the diagram, A, B and C represent three locations. The bearing of B from A is 210^0 and the bearing of C from B is 150^0. Given that /BA/ = 50km and /BC/ = 80km, calculate:

(i) The distance between A and C correct to the nearest kilometer

(ii) The bearing of A from C to the nearest degree.

(b) How far east of B is C?

WASSCE, Nov 1999. No 9 (WAEC).

(5) T

58^0 N

161^0

053^0 B

N 15m

18m

A

N

C

In the diagram, three points A, B and C is on the same horizontal ground. B is 15m from A, on a bearing of 053^0. C is 18m from B on a bearing of 161^0. A vertical pole with top T is erected at B such that angle ATB = 58^0. Calculate, correct to three significant figures,

(a) The length of AC;

(b) The bearing of C from A;

(c) The height of the pole BT.

WASSCE, June 2001, N0 12. (WAEC)

(3) Two planes left Lagos international airport at the same time. The first traveled on a bearing of 048^0 at an average speed of 500km/h for $1^2/_5$ hour before landing. The second traveled on a bearing of 332^0 at an average speed of 400km/h for ¾ hour before landing at its destination.

(a) How far apart are their destinations?

(b) What is the bearing of the first from the second?

WEEK 7

TOPIC: VECTORS

CONTENT:

(a) Vectors as directed line segment.

(b) Cartesian components of a vector.

(c) Magnitude of a vector, Equal vectors, Addition and subtraction of vectors, zero vectors, parallel vectors, multiplication of a vector by a scalar.

Vectors as directed line segment

A vector is any quantity which has direction as well as magnitude or size. Displacement, velocity, force, acceleration are all examples of vectors.

B

A

Since the points are on a Cartesian plane, AB can also be written as a column matrix, or column vector:

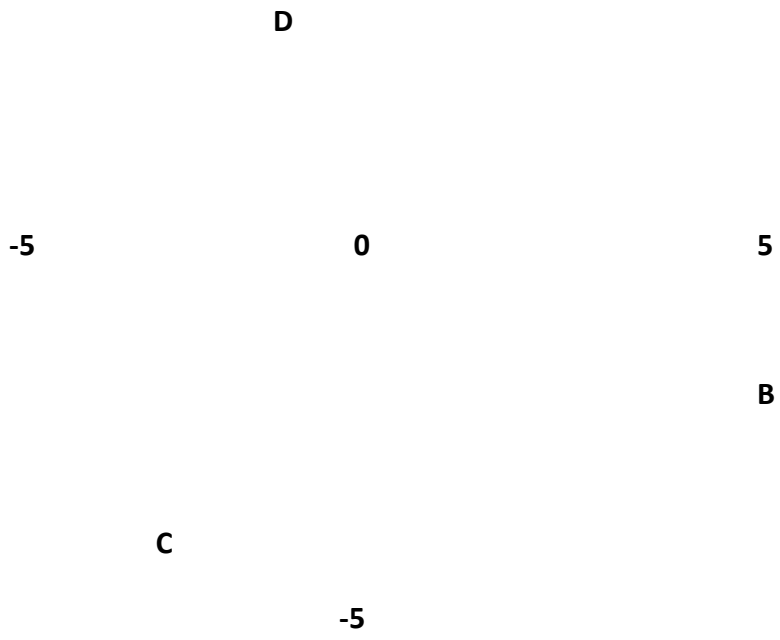
AB = a = , Direction is important. BA is in the opposite direction to AB, although they are both parallel and have the same size:

A displacement vector is a movement in a certain direction without turning.

The vector 'a' is called the position vector of AB

Hence if a point has coordinates (x , y), its position vector is . The figure shows the position vectors

5 **A**



In the figure above, the position vectors are as follows:

Class Activity:

Draw line segments to represent the following vectors.

1.
2.

The component of a vector in the Cartesian plane is denoted by , given the component the 'a' is the i-component of the x-axis while the 'b' is the j-component of the y-axis.

Magnitude of a vector;

If , then , where is the magnitude of a. Notice that the magnitude of a vector is always given as a positive number of units.

Class Activity:

Find the magnitudes or modulus of the following vectors;

-
-
-
-
-

Equal vectors and parallel vectors;

Two or more vectors are equal and parallel if they have the same magnitude and direction. **B**

D

A

C

In the figure above, i.e they are parallel.

Addition and subtraction of vectors;

Vectors are said to be added or subtracted component wise. A vector can be added or subtracted from another if they have equal number of components.

Examples:

1. **Given the vectors , find (i) u + v (ii) u – v (iii) v – u**

Solution:

- **u + v = =**
- **u – v =**
- **v – u =**

2. **Given that A = (3,4) and B = (7,-24), find (i) the addition of A and B (ii) subtract B from A (iii) subtract A from B.**

Class Activity:

1. **If , find;**

- **x – y**
- **x – y + z**
- **z – x – w**
- **w – y + x**
- **(x + y) – (z – w)**

2. **Draw OP = OQ =**

- **Use your drawing to find PQ**
- **Use any method to find OQ – OP**

Multiplication of a vector by a scalar.

If any vector is multiplied by a scalar, say 3, the result is a vector 3 times as big as the initial vector. Also, if multiplied by a scalar, say , the result is a vector half its initial size. Note: A scalar is simply a numerical multiplier.

Examples:

Given the following vectors; AB = CD= , find (i) 2AB (ii) 3BA (iii)

Solution:

- **2AB =**
- **Note that BA = AB,**

3BA =

- **. (note that the entries of a vector can also be in fractional form or decimal)**

Class Activity:

1. **Given that , express each of the following as column vectors,**

- **2a + 3b**

- **– 2b – 5a**
-
-

2. **What is the resultant of the vectors**

PRACTICE EXERCISE:

1. **If**
2. **What is the sum of**
3. **If PQ = u and PR = v, find PM where M is the mid-point of QR.**
4. **A vector is such that**
5. **The coordinates of the vertices of a parallelogram QRST are Q(1,6), R(2,2), S(5,4) and T(x,y).**

- **Find the vectors QR and TS and hence determine the values of x and y.**
- **Calculate the magnitudes of RS and QT.**

WEEK 8

TOPIC: STATISTICS 1

CONTENT:

(a) Meaning and computations of mean, median and mode of ungrouped data.

(b) Determination of the mean, median and the mode of grouped frequency data.

(c) Comparison of mean, mode and median.

(d) Rate and mixtures.

Meaning and computation of mean of ungrouped data

The mean, median and the mode are called measures of central tendency or measures of location. The mean is also known as the average, the median is the middle number while the mode is the most frequent element or data.

THE ARITHMETIC MEAN:

The arithmetic mean is the sum of the ungroup of items divided by the number of it. The mean of an ungrouped data can be calculated by using the formula;

, (when is small) (where the symbol is called sigma meaning summation of all the given data)

Also, Mean, (when is large)

Sum of the product of scores and their corresponding frequencies

Sum of the frequencies

Example 1:

Find the arithmetic mean of the numbers 42, 50, 59, 38, 41, 86 and 56

Solution: Add all the numbers and divide by 7

Example 2:

The table below gives the frequency distribution of marks obtained by some students in a scholarship examination.

Scores(x)	15	25	35	45	55	65	75
Frequency	1	4	12	24	18	8	3

Calculate, correct to 3 significant figures the mean mark of the distribution (WAEC)

Solution:

Scores()	Frequency	
15	1	15

25	**4**	**100**
35	**12**	**420**
45	**24**	**1080**
55	**18**	**990**
65	**8**	**520**
75	**3**	**225**

Since Mean;

(3s.f)

Method 2: mean;

(3s.f)

Example 3:

The table below shows the scores of some students in a quiz

Scores	**1**	**2**	**3**	**4**	**5**	**6**
frequency	**1**	**4**	**5**		**2**	**2**

If the mean score is 3.5, calculate the value of .

Solution:

1	**1**	**1**
2	**4**	**8**
3	**5**	**15**
4		
5	**2**	**10**
6	**2**	**12**

Since, mean

But,

⇒

On cross multiplying

Example 4:

The table below shows the mark distribution of an English language test in which the mean mark is 3. Find the value of.

Mark (x)	**1**	**2**	**3**	**4**	**5**
Frequency(f)	**y**	**3**	**y+3**	**3**	**4 –y**

Solution:

Mean;

1	**Y**	**y**
2	**3**	**6**
3	**y+3**	**3y + 9**
4	**3**	**12**
5	**4 –y**	**20 –5y**

But, mean;

So we have that,

On cross multiplying

Class Activity:

The table below shows the frequency distribution of marks obtained by a group of students in a test. If the mean is 5, calculate the value of x.

Marks	**3**	**4**	**5**	**6**	**7**	**8**
frequency	**5**	**x –1**	**X**	**9**	**4**	**1**

Meaning and computation of median of ungrouped data

The median is the value of the middle item when the items are arranged in order of magnitude either ascending or descending order.

Example 1;

Find the median of the following set of numbers; 16, 13, 10, 23, 36, 9, 8, 48, 24

Solution: Arrange in (either ascending or descending order)

8, 9, 10, 13, 16, 23, 24, 36, 48

The middle number is 16

<u>Median from frequency distribution</u> (i.e when is large)

Median = , when N is odd

Median = when N is even

Example 2:

The table below shows the distribution of marked scored by some students in a maths test

Marks %	**22**	**24**	**36**	**42**	**45**	**48**	**56**	**60**
Frequency	**11**	**2**	**7**	**13**	**10**	**3**	**9**	**5**

Solution:

To find the median, a cumulative frequency table is needed.

Marks %(x)	**Frequency**	**Cumulative frequency**
22	**11**	**11**
24	**2**	**13**
36	**7**	**20**
42	**13**	**33**
45	**10**	**43**
48	**3**	**46**
56	**9**	**55**
60	**5**	**60**

From the table, there are 60 members as indicated by the cumulative frequency.

Since 60 is even, Median =

=

=

The 30th member is 42% and the 31st member is 42%

Example 3:

Calculate the median age from the following data

Age(yrs)	**10**	**12**	**13**	**14**	**16**	**17**	**18**	**19**
No of students	**7**	**15**	**11**	**7**	**12**	**9**	**4**	**6**

Solution:

Ages (yrs)	**No of students**	**Cumulative frequency**
10	**7**	**7**
12	**15**	**22**
13	**11**	**33**
14	**7**	**40**
16	**12**	**52**
17	**9**	**61**
18	**4**	**65**
19	**6**	**71**

Since 71 is odd,

Median = member

=

=

= 36th member

The 36th member falls within the cumulative frequency of up to 40 and this is under 14 years.

Class Activity:

Calculate the median of the distribution below;

Marks (x)	**10**	**20**	**30**	**40**	**50**
Frequency (f)	**13**	**18**	**34**	**60**	**10**

Meaning and computation of mode of ungrouped data

The mode of a given data is the item which occurs most often in the distribution

Example 1;

The record of the marks scored by a number of students in an oral test in economics is as follows;

10, 10, 5, 9, 15, 10, 20, 10, 9, 5, 9, 10, 25, 9, 5, 25. Find the modal mark

Solution:

Marks	**5**	**9**	**10**	**15**	**20**	**25**
Frequency	**3**	**4**	**5**	**1**	**1**	**2**

From the table above, the highest frequency is 5 and this corresponds to a mark of 10

the mode is 10

Example 2;

For a class of 30 students, the scores on a maths test out of 20 marks were as follows

8 10 14 4 6 12 10 10 16 18

10 8 4 6 14 18 16 14 14 14

6 8 10 10 4 6 12 14 14 4

Marks	Frequency
4	4
6	4
8	3
10	6
12	2
14	7
16	2
18	2

Solution:

The highest frequency is 7; modal score = 14

Class Activity:

Find the mode of the following distributions

Age (years)	13	14	15	16	17	18
Frequency	3	10	15	21	5	5

1. **Which of the following is not a measure of central tendency?**

- **Mode**
- **Range**
- **Mean**
- **Median**

2. **The table below shows the distribution of test scores in a class**

Scores (x)	no of pupils
1	1
2	1
3	5
4	3

5	
6	**0**
7	**6**
8	**2**
9	**3**
10	**4**

If the mean score of the test is 6, find the (a) values of k (b) median score

Mean Of Grouped Data

Mean for grouped data can be calculated in two ways;

- **Mean for problems <u>without assumed mean</u>**

where is the class mark or class midpoint

- **Mean of problems <u>with assumed mean</u>**

, where = assumed mean; = deviation from mean ()

Example;

The weights to the nearest kilogram of a group of 50 students in a college of technology are given below:

65 70 60 46 51 55 59 63 68 53 47 53 72 58 67 62 64 70 57 56 73 56 48 51 58

63 65 62 49 64 53 59 63 50 48 72 67 56 61 64 66 52 49 62 71 58 53 69 63 59

- **Prepare a grouped frequency table with class intervals 45–49, 50–54, 55–59 etc**
- **Without the method of assumed, calculate the mean of the grouped data correct to one decimal place.**
- **Using an assumed mean of 62, calculate the mean of the grouped data, correct to one decimal place. (WAEC)**

Solution:

- **Class interval frequency**

45 – 49	**6**
50 – 54	**9**
55 – 59	**10**
60 – 64	**12**
65 – 69	**7**
70 – 74	**6**

- **Mean;**

Class interval	**Class mark(x)**	**frequency (f)**	**fx**
45 – 49	**47**	**6**	**282**
50 – 54	**52**	**9**	**468**
55 – 59	**57**	**10**	**570**
60 – 64	**62**	**12**	**744**
65 – 69	**67**	**7**	**469**
70 – 74	**72**	**6**	**432**

- **, where**

but A = 62,	**Class interval**	**Class mark(x)**	**frequency (f)**	
45 – 49	**47**	**6**	**-15**	**-90**
50 – 54	**52**	**9**	**-10**	**-90**
55 – 59	**57**	**10**	**-5**	**-50**
60 – 64	**62**	**12**	**0**	**0**
65 – 69	**67**	**7**	**5**	**35**
70 – 74	**72**	**6**	**10**	**60**

k

.

Class Activity:

The table below gives the masses in kg of 35 students in a particular school. (NECO)

45 43 54 52 57 59 65 50 61 50 48 53 61 66 47 52 48 40

44 60 68 51 47 51 41 50 62 70 58 42 51 49 55 71 60

- **Using the above given data, construct a group frequency table with class interval 40 – 44, 45 – 49, 50 – 54 etc**
- **From the data above, calculate the mean of the distributions**
- **Using assumed mean of 52, calculate correct to two decimal places the mean of the distribution**

The median of a grouped data

The median formula for grouped data is given as;

Median =

Where; lower class boundary of the median class

n = total frequency

= cumulative frequency before the median class

= frequency of the median class

= size of the median class

Example 1;

The table below shows the marks obtained by forty pupils in a mathematics test

Marks	0 – 9	10 – 19	20 –29	30 – 39	40 – 49	50 – 59
No of pupils	4	5	6	12	8	5

Calculate the median of the distribution.

Solution:

Marks	Class boundaries		
0 – 9	0 – 9.5	4	4
10 – 19	9.5 – 19.5	5	9
20 – 29	19.5 – 29.5	6	15
30 – 39	29.5 – 39.5	12	27
40 – 49	39.5 – 49.5	8	35
50 – 59	49.5 – 59.5	5	40

Median =

20^{th} member

We find the class interval where the median lies, with the aid of the cumulative frequency 20 lies in the after 15. i.e class interval 30 – 39

Median =

=

=

= 29.5 + (0.147 x 10)

= 29.5 + 4.17

= 33.67

Therefore, median mark = 33.67

Class Activity:

1. **The frequency distribution shows the marks of 100 students in a mathematics test.**

Marks	No of students
1 – 10	2
11 – 20	4
21 – 30	9
31 – 40	13
41 – 50	18
51 – 60	32
61 – 70	13
71 – 80	5
81 – 90	3
91 – 100	1

Calculate the median mark. (WAEC)

2. The table below shows the weight distribution of 40 men in a games village.

Weight(kg)	110 – 118	119 – 127	128 – 136	137 – 145	146 – 154	155 – 163	164 – 172
frequency	9	3	4	5	2	5	12

Calculate the median of the distributions

The mode of grouped data

Mode formula for grouped data is given as;

Mode =

Where, Lower class boundary of the modal class

Difference between the modal frequency and the frequency of the next lower class i.e class before it

Difference between the modal frequency and the frequency of the next highest class i.e class after it

Size of the modal class

Example 1:

The table below shows the weekly profit in naira from a mini – market

Weekly profit	1 – 10	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60
frequency	6	6	12	11	10	5

What is the modal weekly profit?

Solution:

Weekly profit	Class boundaries	Frequency
1 – 10	0.5 – 10.5	6

11 – 20	**10.5 – 20.5**	**6**
21 – 30	**20.5 – 30.5**	**12**
31 – 40	**30.5 – 40.5**	**11**
41 – 50	**40.5 – 50.5**	**10**
51 – 60	**50.5 – 60.5**	**5**

The modal class is 21 – 30 (i.e class with the highest frequency)

Mode = ,

Mode =

=

=

=

= 29.07

Modal profit is #29.07

Example 2:

The frequency distribution of the weights of 100 participants in a women conference held in Jupiter is shown below.

Weight(kg)	**40 – 49**	**50 – 59**	**60 – 69**	**70 – 79**	**80 – 89**	**90 – 99**	**100 – 109**
No of women	**9**	**2**	**22**	**30**	**17**	**4**	**16**

Calculate the modal weight of the women

Solution:

Weights (kg)	**Class boundaries**	**No. of women (f)**
40 – 49	**39.5 – 49.5**	**9**
50 – 59	**49.5 – 59.5**	**2**
60 – 69	**59.5 – 69.5**	**22**
70 – 79	**69.5 – 79.5**	**30**

80 – 89	**79.5 – 89.5**	**17**
90 – 99	**89.5 – 99.5**	**4**
100 – 109	**99.5 – 109.5**	**16**

Modal class = 70 – 79;

Mode =

= 69.5 +

= 69.5 +

= 69.5 + 0.381 x 10

= 69.5 + 3.81

= 73.31

Modal weight = 73.3kg (3s.f)

Class Activity:

The table below shows the age distributions of the members of a club.

Age (years)	**10 – 14**	**15 – 19**	**20 – 24**	**25 – 29**	**30 – 34**	**35 – 39**
frequency	**7**	**18**	**25**	**17**	**9**	**4**

Calculate the modal age. (WAEC)

PRACTICE EXERCISE:

1. **If 8kg of coffee costing #2000 a kg is mixed with 12kg of another kind of coffee costing #2200 a kg, what is the cost of the mixture per kg?**
2. **Three kinds of tea at #1,160, #1,460 and #1,540 per kg are in the ratio 2:3:5. What is the mixture worth per kg.**
3. **Four ingredients costing #320 per kg, #240 per kg, #160 per kg and #80 per kg are mixed so that their masses are in ratio 4:1:2:3. Calculate the average cost per kg of the mixture.**
4. **A trader mixes three bags of sugar costing #900/bag with seven sacks of sugar which cost #700/bag. If she sells the mixture at #950/bag, calculate her percentage profit.**

5. **A trader bought three kinds of nuts at #100 per kg, #84 per kg and #60 per kg respectively. He mixed them in the ratio 3:5:4 respectively and sold the mixed nuts to make a profit of 25%. At what price per kg did sell them?**

ASSIGNMENT:

1. **The marks scored by 30 students in a particular subject are as follows;**

39 31 50 18 51 63 10 34 42 89 73 11 33 31 41

25 76 13 26 23 29 30 51 91 37 64 19 86 9 20

- **Prepare a frequency table, using class intervals 1 – 20, 21 – 40 e.t.c**
- **Calculate the mean mark**
- **Calculate the modal score**

2. **The table below shows the monthly profit in #100,000 of naira of a super market**

Monthly profit in #100,000	**11 – 20**	**21 – 30**	**31 – 40**	**41 – 50**	**51 – 60**	**61 – 70**
						8
frequency	**5**	**11**	**9**	**10**	**7**	

- **What is the modal monthly?**
- **Estimate the mean and the median profit**

WEEK 9

TOPIC: STATISTICS 3

CONTENT:

Histograms of grouped data (Revision): (a) Need for grouping (b) Calculation of; (i) class boundaries (ii) class interval (iii) class mark. (b) Frequency polygon (c) Cumulative Frequency graph: (a) Calculation of cumulative frequencies. (b) Drawing of cumulative frequency curve graph (Ogive). (c) Using graph of cumulative frequencies to estimate; (i) Median (ii) Quartiles (iii) Percentiles. (iv) Other relevant estimates. (d) Application of ogive to everyday life.

Let the record below be the mass of some people (in kg)

66	**48**	**71**	**61**	**39**	**68**	**33**	**60**	**52**	**44**
33	**49**	**81**	**58**	**59**	**71**	**42**	**88**	**68**	**91**
80	**66**	**70**	**26**	**96**	**63**	**76**	**46**	**51**	**61**
54	**32**	**50**	**59**	**41**	**55**	**38**	**56**	**86**	**62**
50	**69**	**23**	**84**	**77**	**33**	**71**	**42**	**69**	**93**

Should bar chart be drawn for the different masses above, there would be too many bars, so the data may be grouped into class intervals and then a frequency distribution table prepared. Appropriate class intervals are : 21 – 30, 31 – 40, 41 – 50, …

Each data belongs to one of the class intervals. Each data is first represented by a stroke in the tally column. Every fifth stroke is used to cross the first four counted. The number of tally in each class interval gives the frequency

Class interval	**Tally**	**Frequency**
21 – 30	**//**	**2**
31 – 40	**//// /**	**6**
41 – 50	**//// ////**	**9**
51 – 60	**//// ////**	**9**
61 – 70	**//// //// /**	**11**
71 – 80	**//// /**	**6**

81 – 90	**////**	**4**
91 – 100	**///**	**3**

The modal class is the one with the highest frequency.

Class Activity:

1. **Prepare a frequency table, using class intervals 1 - 20, 21 – 40, … for the scores by 30 students.**

26	**23**	**29**	**30**	**91**	**51**
37	**64**	**86**	**9**	**20**	**19**
39	**31**	**50**	**18**	**51**	**63**
33	**13**	**31**	**25**	**41**	**76**
10	**34**	**42**	**89**	**73**	**11**

2. **The marks scored by fifty students in an examination paper are given below:**

43	**27**	**31**	**43**	**22**	**31**	**47**	**34**	**18**	**15**
30	**45**	**48**	**55**	**39**	**25**	**31**	**12**	**18**	**21**
26	**19**	**38**	**10**	**44**	**43**	**51**	**33**	**59**	**54**
41	**35**	**37**	**41**	**46**	**33**	**51**	**37**	**48**	**58**
17	**19**	**23**	**26**	**29**	**38**	**57**	**36**	**35**	**44**

Prepare a frequency table, using class intervals 10 – 19, 20 – 29, 30 – 39, e.t.c

What is the modal class?

Calculation of (i) class boundaries

(ii) class interval

(iii) class mark

Grouped data can be represented using a kind of rectangles called histogram. The width of these rectangles is determined by the class interval while the height is proportional to the frequency in that interval. To close up the gaps between the class intervals, the class interval at both ends to have a common boundary in-between two intervals. From the last frequency table above we get this table.

Class intervals	Frequency	Class boundaries
21 – 30	2	20.5 – 30.5
31 – 40	6	30.5 – 40.5
41 – 50	9	40.5 – 50.5
51 – 60	9	50.5 – 60.5
61 – 70	11	60.5 – 70.5
71 – 80	6	70.5 – 80.5
81 -90	4	80.5 – 90.5
91 – 100	3	90.5 – 100.5

To get a common boundary between two class interval, the upper class limit of a class is added to the lower class limit of the next class and divide the sum by 2.

e.g

e.t.c

The upper class boundary of a class is the lower class boundary of the next class. This gives a continuous horizontal axis.

Another thing to consider is the class mark or class centre. This may be used in finding the mean. For any class interval, the class center is the average of the upper and lower limits of that particular class interval.

Class center of interval 21 – 30 is

Class mark for class interval 31 – 40 is

The class mid-values (class centre) are used in plotting frequency polygon.

CUMULATIVE FREQUENCY GRAPH

The Cumulative frequency of a given class or group is the sum of the frequency of all the classes below and including the class itself.

Cumulative frequency curve or Ogive is a statistical graph gotten by plotting the upper class boundaries against cumulative frequencies. It is used to determine among the others: Median, Percentiles (100 divisions), Deciles (10 divisions), Quartiles (4 divisions)

The cumulative frequencies are placed along the y – axis, while the scores or class boundaries are placed along the x-axis

Calculation of cumulative frequencies and Drawing of cumulative frequency curve graph (Ogive)

Example 1;

The table below shows the frequency distributions of the lengths (in cm) of fifty planks cut by a machine in the wood – processing factory of kara sawmill (Nigeria)

Class interval	21 – 30	31 – 40	41 – 50	51 – 60	61 – 70	71 – 80	81 – 90	91 – 100
frequency	2	6	9	9	11	6	4	3

- **Prepare a cumulative frequency table for the distribution**
- **Draw the cumulative frequency curve (Ogive) for the distribution**

Scale: 2cm to represent 10 units on the frequency axis

2cm to represent 10 units on the length axis

Solution:

The cumulative frequency table is given below as;

Class interval	Class boundaries	Frequency	Cumulative frequency
21 – 30	20.5 – 30.5	2	2
31 – 40	30.5 – 40.5	6	6 + 2 = 8

41 – 50	**40.5 – 50.5**	**9**	**9 + 8 = 17**
51 – 60	**50.5 – 60.5**	**9**	**9 + 17 = 26**
61 – 70	**60.5 – 70.5**	**11**	**11 + 26 = 37**
71 – 80	**70.5 – 80.5**	**6**	**6 + 37 = 43**
81 – 90	**80.5 – 90.5**	**4**	**4 + 43 = 47**
91 – 100	**90.5 – 100.5**	**3**	**3 + 47 = 50**

To plot the graph, it is advisable to use a suitable scale. The graph should be drawn big, because the bigger the graph the more accurate the answers that would be obtained from the graph.

Cumulative frequency curve

Using graph of cumulative frequencies to estimate median, quartiles, percentiles etc

To estimate median and quartiles from the Ogive or cumulative frequency curve, we take the following steps;

STEP 1: Compute to find their position on the cumulative frequency (CF) axis using the following formulae,

(a) For lower quartile or first quartile () we use

(b) For median quartile or second quartile (), we use

(c) For upper quartile or third quartile (), we use (Total frequency or last CF)

Cumulative frequency

Upper class boundaries

STEP 2: Locate the point on the cumulative frequency axis and draw a horizontal line from this point to intersect the Ogive.

STEP 3: At the point it intersect the Ogive, draw a line parallel to the cumulative frequency axis to intersect the horizontal axis.

STEP 4: Read the value of the desired quartile at the point of intersection of the vertical line and the horizontal axis.

Inter-quartile range =

Semi inter-quartile range

Percentile

This is the division of the cumulative frequency into 100 points. For instance;

75% =

20% =

Then, we trace the required values to the graph (curve) then to the class boundaries to get the required answer.

Example 1:

Weight (kg)	**20 – 29**	**30 – 39**	**40 – 49**	**50 – 59**	**60 – 69**	**70 –79**
No of participants	**10**	**18**	**22**	**25**	**16**	**9**

The frequency distribution of the weight of 100 participants in a high jump competition is as shown below:

- **Construct the cumulative frequency table**
- **Draw the cumulative frequency curve**
- **From the curve, estimate:**
- **The median**
- **The lower quartile**

- **The upper quartile**
- **The inter-quartile range**
- **The semi inter-quartile range**
- **65 percentile**
- **4^{th}decile**
- **The probability that a participant chosen at random weighs at least 60kg**

Solution:

Class interval	Class boundary	Frequency	Cumulative Frequency
20 – 29	19.5 – 29.5	10	10
30 – 39	29.5 – 39.5	18	28
40 – 49	39.5 – 49.5	22	50
50 – 59	49.5 – 59.5	25	75
60 – 69	59.5 – 69.5	16	91
70 – 79	69.5 – 79.5	9	100

(b)

(c i.) From the curve, median is half way up the distribution. This is obtained by using where N is the total frequency. Median = =

Median is at point on the graph, i.e median = 49.5kg

1. **Lower quartile is one-quarter of the way up the distribution; lower quartile = = = 25**

25th position

Lower quartile is at point on the graph. i.e lower quartile = 37.5kg

iii. Upper quartile is three-quarters way up the distribution;

Upper quartile =

=

=

= 75th position

Upper quartile is at the point on the graph. i.e Upper quartile = 59.5kg

1. **Inter-quartile range (IQR) = Upper quartile – Lower quartile**

=

= 59.5kg – 37.5kg

= 22kg

1. **Semi inter-quartile range (SIQR) =**

=

SIQR = 11kg

1. **65 percentile =**

=

= 65^{th} position

65 percentile is at point p on the graph = 54.5kg

vii. 4^{th} deciles =

=

= 40^{th} position

4^{th} deciles is at point d on the graph i.e 44.5kg

viii. Probability of at least 60kg = =

Application of Ogive to everyday life

Example 1;

The table below shows the frequency distribution of the marks of 800 candidates in an examination

Marks	Frequency
0 – 9	10
10 – 19	40
20 – 29	80
30 – 39	140
40 – 49	170
50 – 59	130
60 – 69	100
70 – 79	70

80 – 89 40

90 – 99 20

(ai.) Construct a cumulative frequency table

1. **Draw the Ogive**

iii. Use your Ogive to determine the 50^{th} percentile

(b.) The candidates that scored less than 25% are to be withdrawn from the institution, while those that scored more than 75% are to be awarded scholarship. Estimate the number of candidates that will be retained, but will not enjoy the award

(c.) If 300 candidates are to be admitted out of the 800 candidates for a particular course in the institution, what will be the cut of mark for the admission?

(d.) if a candidate is picked from the population, what is the probability that the candidate scored above 40%?

Solution: (ai.)

Marks (%)	Class Boundary	Frequency	Cumulative frequency
0 – 9	– 0.5 – 9.5	10	10
10 – 19	9.5 – 19.5	40	50
20 – 29	19.5 – 29.5	80	130
30 – 39	29.5 – 39.5	140	270
40 – 49	39.5 – 49.5	170	440
50 – 59	49.5 – 59.5	130	570
60 – 69	59.5 – 69.5	100	670
70 – 79	69.5 – 79.5	70	740
80 – 89	79.5 – 89.5	40	780
90 – 99	89.5 – 99.5	20	800

iii. 50^{th} percentile =

= 400 position

50^{th} percentile is at the point on the graph = 47.5%

(b.) To get the number of candidate that scored less than 25%, we would read from the mark axis at the point of 25% to the frequency axis for the number of candidates.

From the graph, this is at the point number 80. Therefore 80 candidates are to be withdrawn from the institution.

Those that scored more than 75% would also be read from the mark axis to the frequency axis. From the graph, this is 720;

Number of candidates = 800 – 720

= 80 candidates

.: 80 candidates are to be awarded scholarship, the number of candidates that will be retained without award = 800 – (80 + 80)

= 800 – 160 = 640 candidates

(c.) If 300 candidates are to be registered for the course, then the 300 candidates would be obtained from the top of the frequency axis. This is read from the point C on the graph

i.e 800 – 300 = 500 position

The cut-off mark from the graph is 55.5%

(d.) Reading from the mark axis at 40.5%, we get the value 290 from the graph

Those that scored 40% and below = 290 candidates

Those that scored above 40% = 800 – 290 = 510 candidates

Therefore, probability that the candidate scored above 40% = =

ASSIGNMENT:

1. **In the test conducted in a particular school, the students are graded according to the marks scored as given in the table below; this is the scores of 2000 candidates**

Marks (%)	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60	61 – 70	71 – 80	81 – 90
Pupil's no	68	184	294	402	480	310	164	98

- **Prepare a cumulative frequency table and draw the cumulative frequency curve for the distribution.**
- **Use your curve to estimate the; (i) cut off mark if 300 candidates are to be offered admission (ii) probability that a candidate picked at random scored at least 45%**

2. **The table below shows the marks scored by a group of students in a test**

Marks	1 – 10	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60	61 – 70	71 – 80	81 – 90	91 - 100
Frequency	4	6	9	12	20	15	7	5	0	2

- **Construct the cumulative frequency table**
- **Draw the ogive**
- **From your ogive, find the: (i) Median (ii) Lower quartile**
- **A student was picked at random from the group, what is the probability that the students (using o-give) (i) Obtain a distinction grade of 75% and above (ii) failed the test if the pass mark is 40%**

WEEK 10

TOPIC: STATISTICS 2

CONTENT:

(a) Definitions of: (i) Range, (ii) Variance, (iii) Standard deviation.

(b) Calculation of range, variance and standard deviation.

(c) Practical application in capital market reports; (i) Home (ii) Health studies (iii) Population studies.

DEFINITION AND CALCULATION OF RANGE

Measures of Dispersion

The measure of dispersion (also called measure of variation) is concerned with the degree of spread of the numerical value of a distribution.

Range: This is the difference between the maximum and minimum values in the data.

Examples 1:

Find the range of the data 6, 6, 7, 9, 11, 13, 16, 21 and 32

Solution: The maximum item is 32

The minimum item is 6

∴ Range = 32 – 6 = 26

Example 2:

Find the range of the distributions below 65,62,62,61,61,60,60,59,58,52

Solution: Range = 65 – 52 = 13

Deviation from the mean:

If the mean of a distribution is subtracted from any value in the distribution, the result is called the DEVIATION of the value from the mean.

Consider the table below (set of examination marks)

65 62 62 61 61

60 60 59 58 52

The mean =

=

= 60

Deviation from the mean =

= 62 – 60 = +2

= 62 – 60 = +2

= 61 – 60 = +1

= 61 – 60 = +1

= 60 – 60 = 0 e.t.c

The deviations of the scores from the mean are +5, +2, +2, +1, +1, 0, 0, -1, -2, -8

The sum of these deviations = 0

Class Activity:

- **Calculate the range of the following distributions**
- **72, 78, 72, 90, 72, 83, 79.**
- **9, 4.0, 4.2, 3.9, 3.8, 4.0**
- **Calculate the mean deviation of (1a) and (1b) above**

DEFINITION AND CALCULATION OF VARIANCE

The variance is the arithmetic mean of the squares of the deviation of the observations from the true mean. It is also called the mean squared deviation.

The formula for variance is (a) for an ordinary distribution (ungrouped)

(b) , for a frequency distribution table (grouped)

Example 1:

Calculate the variance of the following distributions of the ages of 50 pupils in a secondary school

Age (years)	10	12	13	14	15	16
Number of pupils	18	4	6	12	6	4

Age (x)	Freq (f)				
10	18	180	2.6	6.76	121.68
12	4	48	0.6	0.36	1.44
13	6	78	0.4	0.16	0.96
14	12	168	1.4	1.96	23.52
15	6	90	2.4	5.76	34.56
16	4	64	3.4	11.56	46.24
	50	628			228.4

Mean

= 12.6

Variance =

=

= 4.568

= 4.6 approximately

Example 2:

Calculate the variance of the distribution below.

90, 80, 72, 68, 64, 56, 52, 48, 36, 34

Solution:

Mean

= 60

90	**+30**	**900**
80	**+20**	**400**
72	**+12**	**144**
68	**+8**	**64**
64	**+4**	**16**
56	**-6**	**16**
52	**-8**	**64**
48	**-12**	**144**
36	**-24**	**576**
34	**-26**	**676**
		Total = 3000

Variance =

=

= 300

Class Activity:

Calculate the mean and variance of the ages of 12 students aged 16, 17, 18, 16.5, 17, 18, 19, 17, 17, 18, 17.5 and 16

Definition and Calculation of standard deviation

Standard deviation (S.D) is the square root of variance.

The formula for S.D are: (a) and (b)

Example 1:

Find the variance and standard deviation of the set of numbers 2,5,6,3 and 4

Solution: Variance =

But mean = 4

2	**-2**	**4**
5	**1**	**1**
6	**2**	**4**
3	**-1**	**1**
4	**0**	**0**

Variance = = 2

Standard deviation, S.D =

=

= 1.414

Example 2:

Calculate the standard deviation of the distribution

Age (years)	**10**	**12**	**13**	**14**	**15**	**16**
Frequency	**18**	**4**	**6**	**12**	**6**	**4**

Solution:

Reference to example 2 n page 3 and 4

Standard Deviation =

=

=

= 2.14

Class Activity:

Compute (i) the variance (ii) the standard deviation of the data.

1. **In a college, the number of absentees recorded over a period of 30days was a shown in the frequency distribution table.**

Number of absentees	**0 – 4**	**5 – 9**	**10 – 14**	**15 – 19**	**20 – 24**
Number of days	**1**	**5**	**10**	**9**	**5**

2. **The table shows the distribution of ages of workers in a company**

Age (in yrs)	**17 – 21**	**22 – 26**	**27 – 31**	**32 – 36**	**37 – 41**	**42 – 46**	**47 – 51**	**52 – 56**
Frequency	**12**	**24**	**30**	**37**	**45**	**25**	**10**	**7**

PRACTICAL APPLICATION IN CAPITAL MARKET REPORT

EXAMPLE :

Two groups of eight students in a class were given a test in English. Group A had the following marks; 60, 70, 50, 48, 68, 72, 80 and 56

Group B had the following marks: 50, 90, 40, 58, 90, 82, 60 and 44.

- **Calculate the mean, range, variance and standard deviation of each group.**
- **Which group had less variation in its marks?**

Solution:

- **Group A**

60	**-3**	**3**	**9**
70	**7**	**7**	**49**
50	**-13**	**13**	**169**

48	**-15**	**15**	**225**
68	**+5**	**5**	**25**
72	**+9**	**9**	**81**
80	**+17**	**17**	**289**
56	**-7**	**7**	**49**
			896

Mean

=

= 63

Range = 70 50 = 20

Variance (v) =

=

= 112

S.D =

=

=

= 10.5830

= 10.58 (2 d.p)

GROUP B:

50	**14.25**	**203.0625**

90	**25.75**	**663.0625**
40	**24.25**	**588.0625**
58	**6.25**	**39.0625**
90	**25.75**	**663.0625**
82	**17.75**	**315.0625**
60	**4.25**	**18.0625**
44	**20.25**	**410.0625**
		2899.5

Mean = 64.25

Mean = 64.25

Variance = 362.43

S.D = 19.04 (2 d.p)

(b) Group A

Class Activity:

1. **The rainfall in millimetres from June to November in two towns is given below**

	June	July	Aug	Sept	Oct	Nov
Town A	**1.8**	**2.7**	**1.4**	**2.4**	**2.8**	**1.5**
Town B	**3.4**	**3.6**	**2.2**	**2.5**	**2.8**	**1.2**

- **Compare the means and standard deviations of rainfall in towns A and B**
- **In which town is rainfall less widely spread during the period?**

2. **Compute the (i) Variance**

(ii) Standard deviations

(iii) Range of the following distributions

Score	95	85	80	75	70	65	55	40
frequency	1	1	1	4	1	3	1	3

WEEK 11 REVISION

EXAMINATION